i.i.d. Mixed Inputs and Treating Interference as Noise are gDoF Optimal for the Symmetric Gaussian Two-user Interference Channel

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Interference as Noise: Friend or Foe? [ISIT15 extended version at arXiv:1506.02597]

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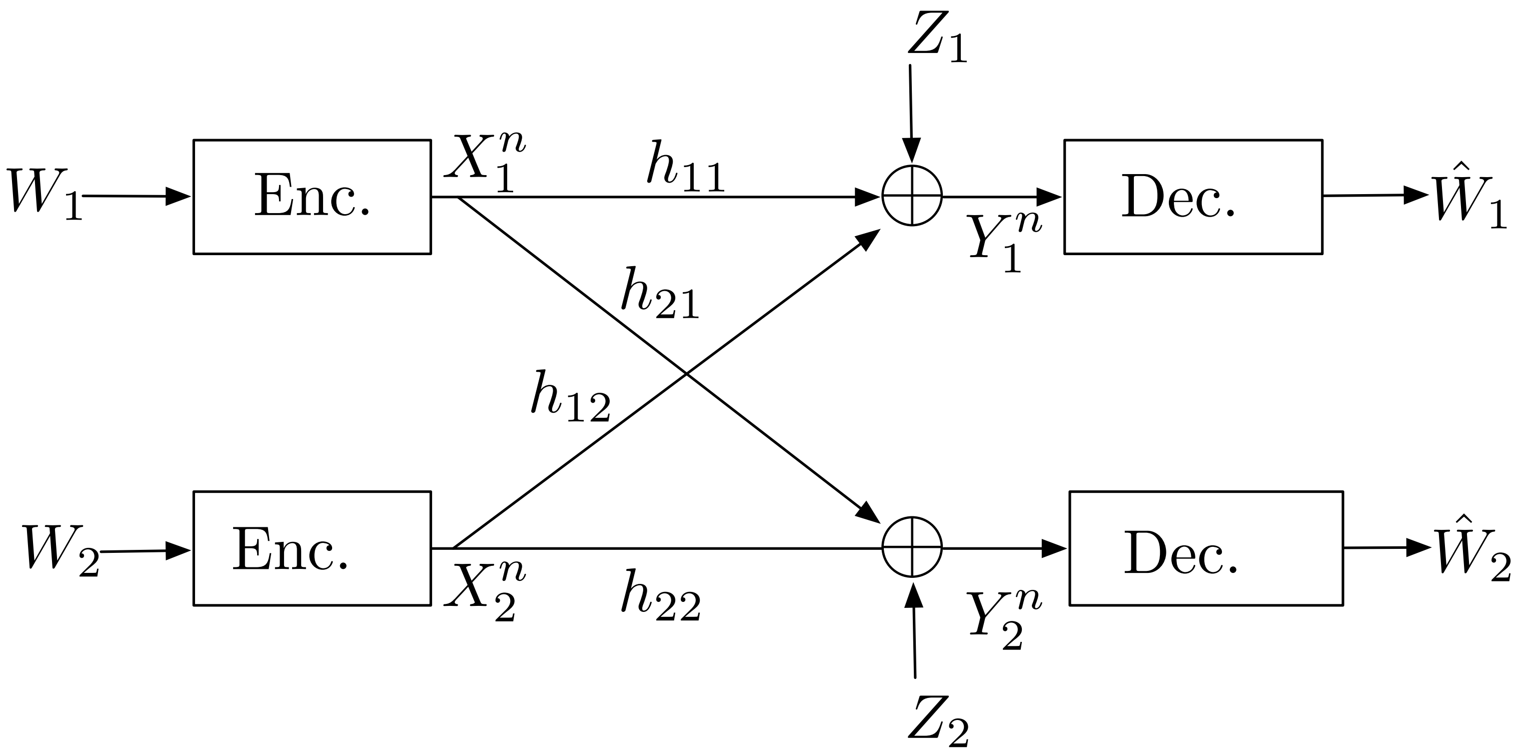
Channel Model



with the usual assumptions and capacity region definition.

Symmetric: $$|h_{11}|^2 = |h_{22}|^2 = \mathrm{S}$$ $$|h_{12}|^2 = |h_{21}|^2 = \mathrm{I}$$

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Motivation

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Capacity: $\mathcal{C} = \lim_{n\to\infty} \mathrm{co}\left(\bigcup_{P_{X_1^n X_2^n}=P_{X_1^n}P_{X_2^n}} \left\{ \begin{array}{l} 0 \le R_1 \le \frac{1}{n} I(X_1^n;Y_1^n) \\ 0 \le R_2 \le \frac{1}{n} I(X_2^n;Y_2^n) \end{array} \right\}\right)$

R. Ahlswede, "Multi-way communication channels," in Proc. IEEE Int. Symp. Inf. Theory, March 1973, pp. 23–52.

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R. Ahlswede, "Multi-way communication channels," in Proc. IEEE Int. Symp. Inf. Theory, March 1973, pp. 23–52.

Treat Interference as Noise Inner Bound:

$\mathcal{R}_{\text{in}}^{\text{TIN+TS}} = \text{co}\left(\bigcup_{P_{X_1 X_2}=P_{X_1}P_{X_2}} \left\{ \begin{array}{l} 0 \le R_1 \le I(X_1;Y_1) \\ 0 \le R_2 \le I(X_2;Y_2) \end{array} \right\}\right)$ With Time Sharing

$\mathcal{R}_{\text{in}}^{\text{TINnoTS}} = \bigcup_{P_{X_1 X_2}=P_{X_1}P_{X_2}} \left\{ \begin{array}{l} 0 \le R_1 \le I(X_1;Y_1) \\ 0 \le R_2 \le I(X_2;Y_2) \end{array} \right\}$ No Time Sharing

Motivation

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With Time Sharing

$$\mathcal{R}_{\mathrm{in}}^{\mathrm{TINnoTS}} = \bigcup_{P_{X_1 X_2}=P_{X_1}P_{X_2}} \left\{ \begin{array}{l} 0 \le R_1 \le I(X_1;Y_1) \\ 0 \le R_2 \le I(X_2;Y_2) \end{array} \right\}$$

No Time Sharing

How far away is TINnoTS from the Capacity?

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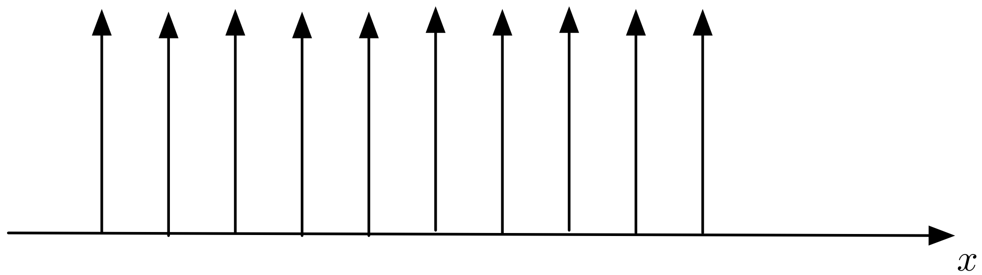
Outline

- Definitions and relevant past work
- Useful tools:
  - Mutual information lower bounds
  - Minimum Distance lower bounds
- Main result

Mixed Inputs

$$X = \sqrt{1-\delta}\; X_D + \sqrt{\delta}\; X_G,$$
$$\delta \in [0,1],$$
$$X_D \sim \mathrm{PAM}\,(N)\,,$$
$$X_G \sim \mathcal{N}(0,1)$$

PAM



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Known Results

Capacity Strong Interference

$$|h_{11}|^2 \leq |h_{21}|^2 \text{ and } |h_{22}|^2 \leq |h_{12}|^2$$

Sato, H., "The capacity of the Gaussian interference channel under strong interference," IEEE Trans. Inf. Theory, vol. 27, no. 6, pp. 786,788, Nov 1981.

Sum-Capacity in Very Weak Interference

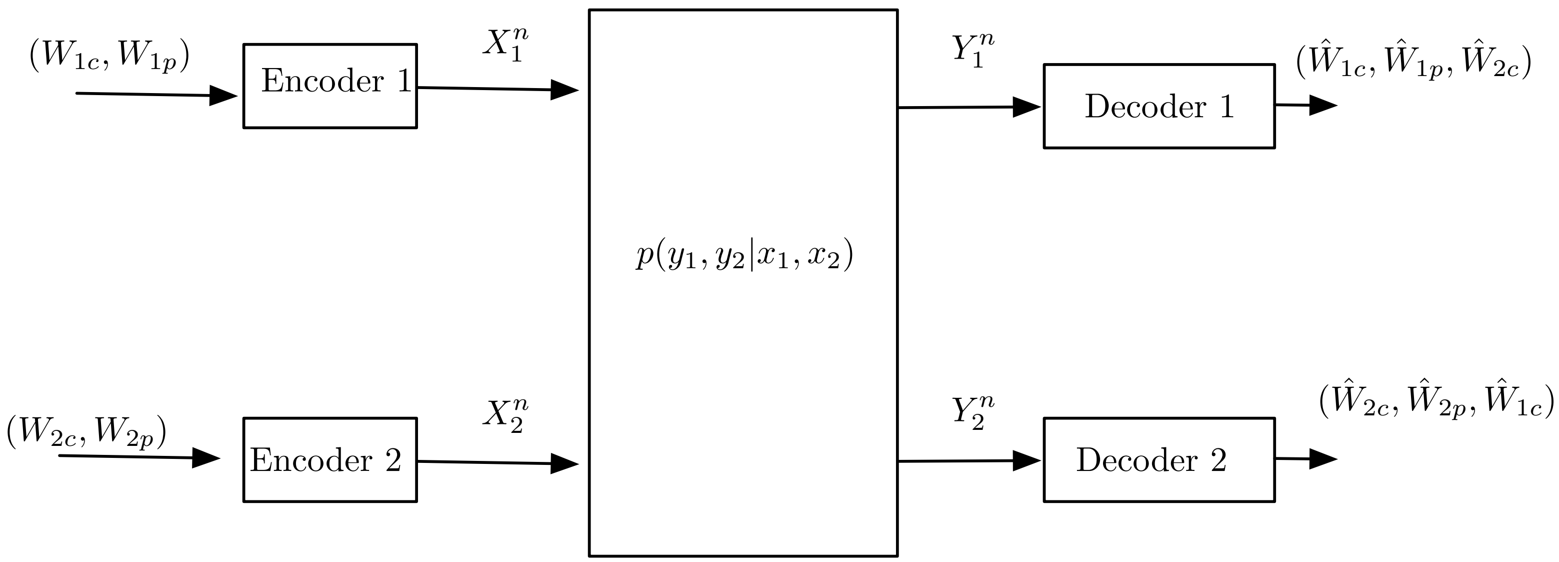
$$\sqrt{\frac{\mathrm{S}}{\mathrm{I}}}(1+\mathrm{I}) \leq \frac{1}{2}$$

X. Shang, G. Kramer, and B. Chen, “A new outer bound and the noisy-interference sum rate capacity for Gaussian interference channels,” IEEE Trans. Inf. Theory, vol. 55, no. 2, pp. 689–699, 2009.

Motahari, A.S.; Khandani, A.K., "Capacity Bounds for the Gaussian Interference Channel," IEEE Trans. Inf. Theory, vol.55, no.2, pp.620,643, Feb. 2009

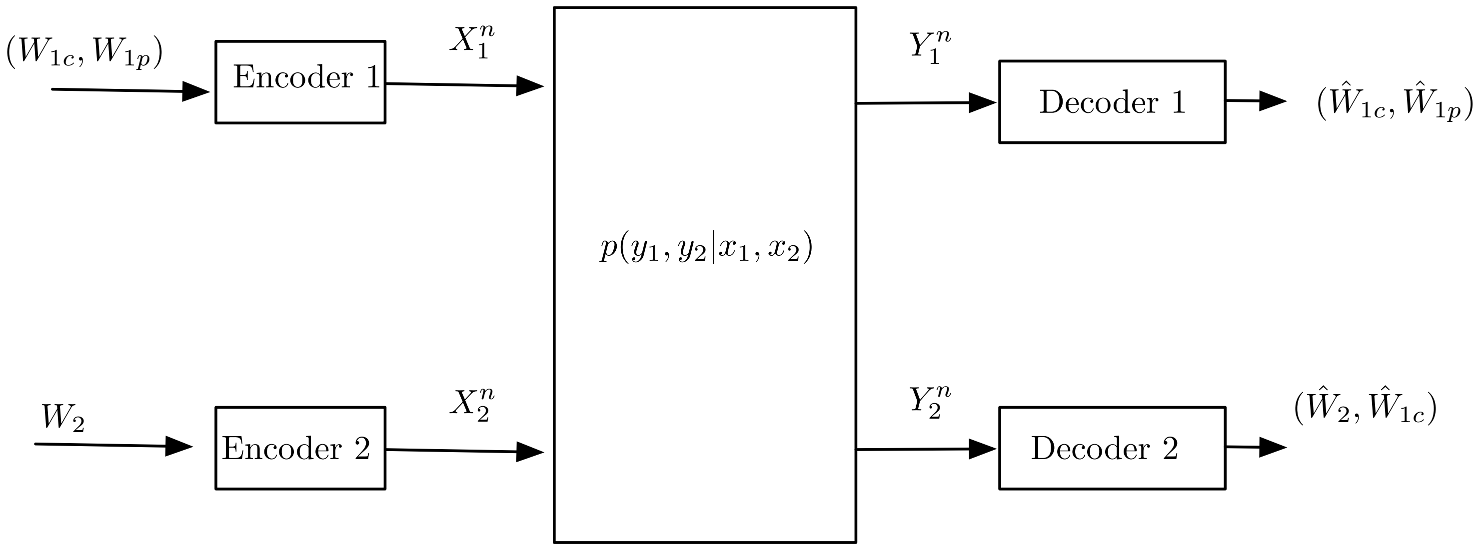
Annapureddy, V.S.; Veeravalli, V.V., "Gaussian Interference Networks: Sum Capacity in the Low-Interference Regime and New Outer Bounds on the Capacity Region," IEEE Trans. Inf. Theory, vol.55, no. 7, pp.3032,3050, July 2009

Approximate Capacity



HK+Gaussian Inputs
1/2 bit

R. Etkin, D. Tse, and H. Wang, “Gaussian interference channel capacity to within one bit,” IEEE Trans. Inf. Theory, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.



“One-sided” HK+ Mixed Inputs
1.75 bits

A. Dytso, D. Tuninetti, and N. Devroye, “On the two-user interference channel with lack of knowledge of the interference codebook at one receiver,” IEEE Trans. Inf. Theory, vol. 61, no. 3, pp. 1257–1276, March 2015.

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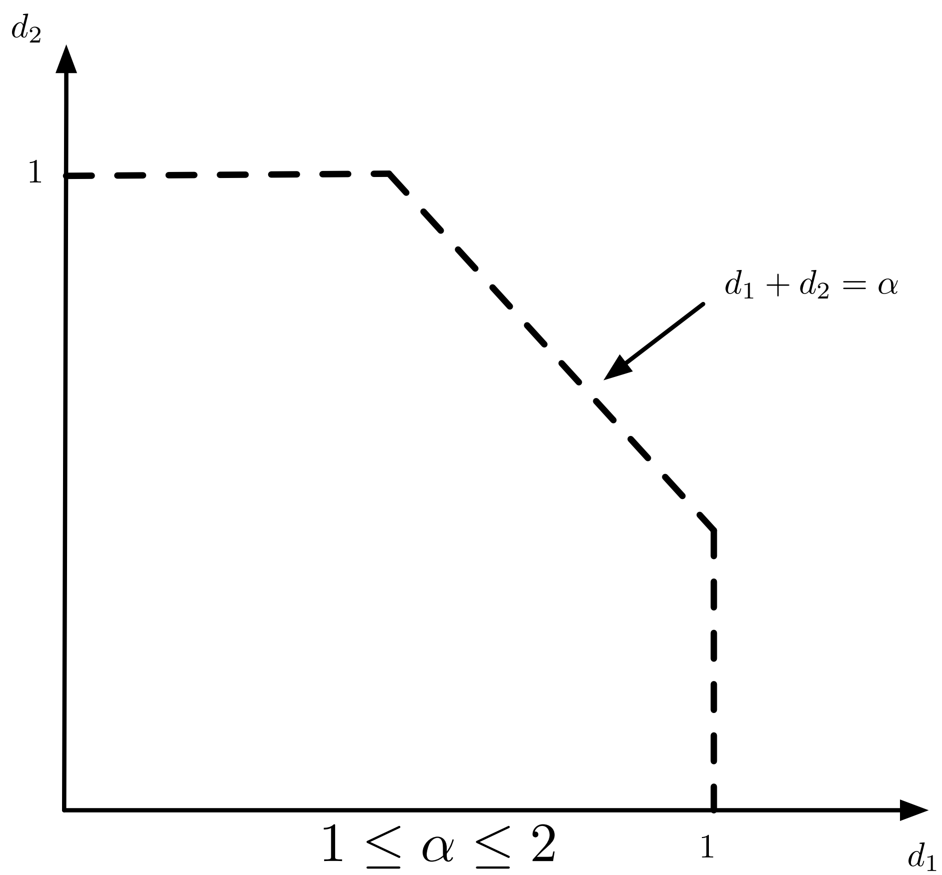
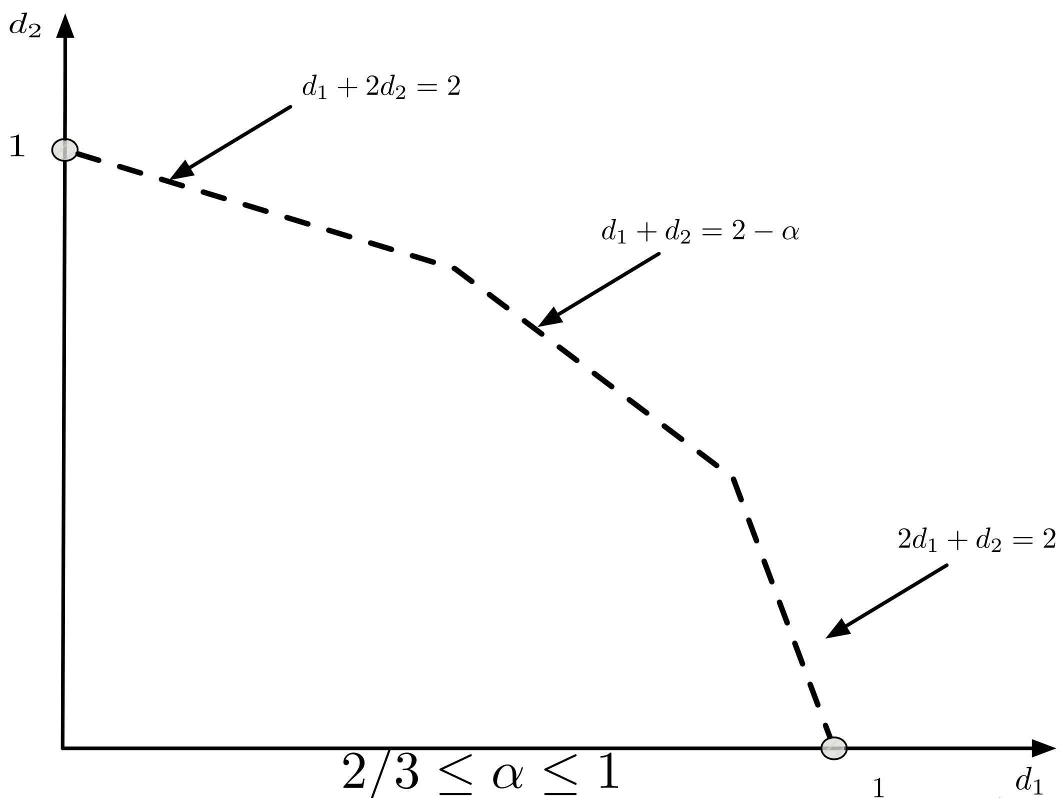
gDoF Region

$$\mathcal{D}(\alpha) := \left\{ (d_1, d_2) \in \mathbb{R}_+^2 : d_i := \lim_{\substack{\mathrm{I} = \mathrm{S}^\alpha, \\ \mathrm{S} \to \infty}} \frac{R_i}{\frac{1}{2}\log(1+\mathrm{S})}, i \in [1:2],\ (R_1, R_2) \text{ is achievable} \right\}.$$



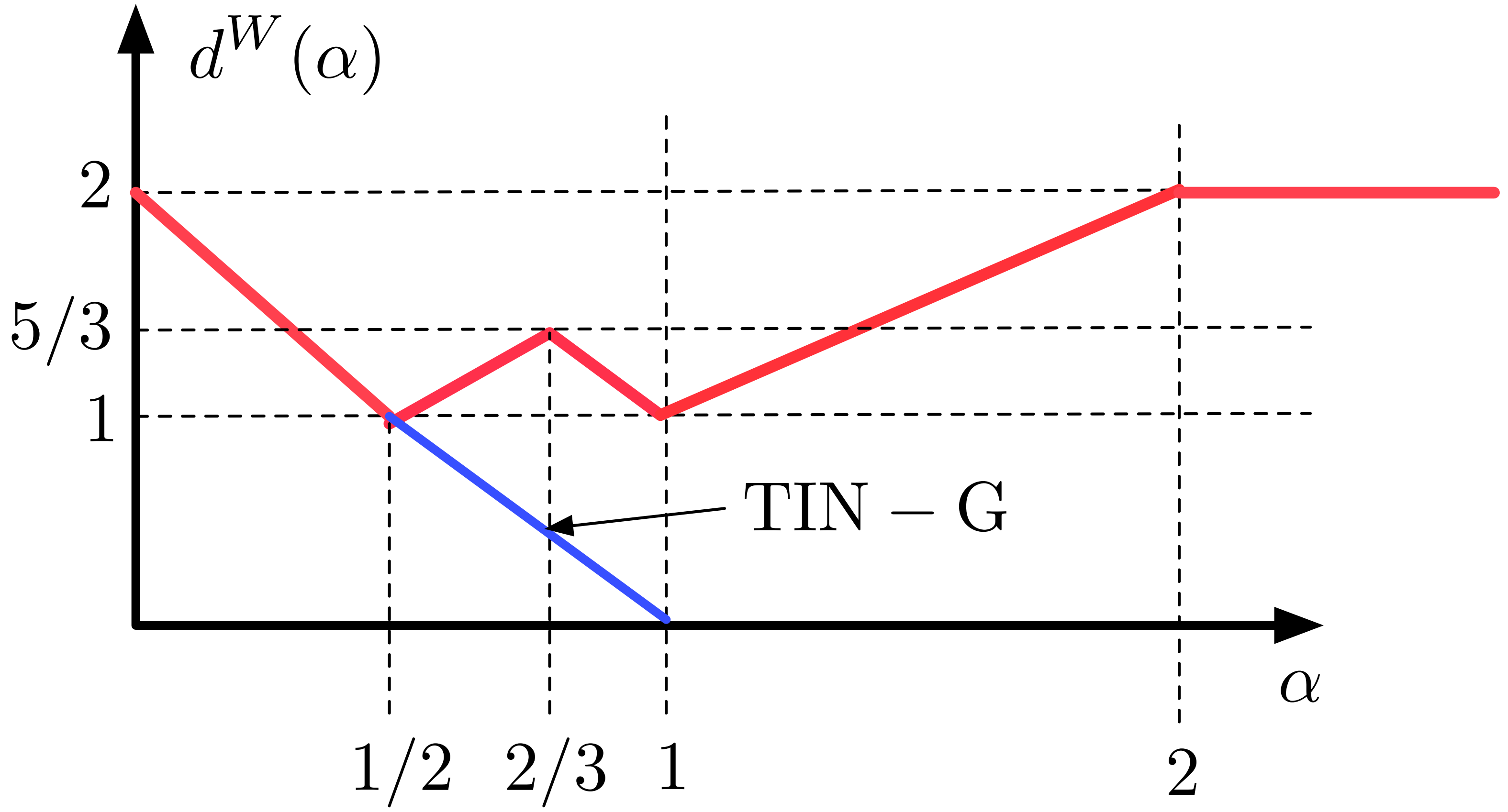


R. Etkin, D. Tse, and H. Wang, “Gaussian interference channel capacity to within one bit,” IEEE Trans. Inf. Theory, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.

Sum-gDoF [ISIT`14]

TINnoTS with mixed inputs is to within O(loglog(SNR)) of the sum-capacity

A. Dytso, D. Tuninetti, and N. Devroye, "On Gaussian interference channels with mixed Gaussian and discrete inputs," in Proc. IEEE Int. Symp. Inf. Theory, June 2014, pp. 261–265.



$$\mathcal{R}_{\text{in}}^{\text{TINnoTS}} = \bigcup_{P_{X_1X_2}=P_{X_1}P_{X_2}} \left\{ \begin{array}{l} 0 \le R_1 \le I(X_1;Y_1) \\ 0 \le R_2 \le I(X_2;Y_2) \end{array} \right\}$$

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Achievability

$$\mathcal{R}_{\text{in}}^{\text{TINnoTS}} = \bigcup_{N_1,N_2,\delta_1,\delta_2} \left\{ \begin{array}{l} 0 \le R_1 \le I(X_1;Y_1) \\ 0 \le R_2 \le I(X_2;Y_2) \end{array} \right\}$$

with

$$\begin{aligned} X_i &= \sqrt{1-\delta_i}\ X_{iD} + \sqrt{\delta_i}\ X_{iG}, \\ &\delta_i \in [0,1], \\ &X_{iD} \sim \text{PAM}\left(N_i\right), \\ &X_{iG} \sim \mathcal{N}(0,1), \\ &i = 1,2. \end{aligned}$$

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Analytical rate expressions not trivial

$$\begin{aligned} I(X_2; Y_2) &= I(X_2; h_{21} X_1 + h_{22} X_2 + Z_G) \\ &= I\left(X_{1D}, X_{2D}; \frac{\sqrt{1-\delta_1} h_{21} X_{1D} + \sqrt{1-\delta_2} h_{22} X_{2D}}{\sqrt{1 + |h_{21}|^2 \delta_1 + |h_{22}|^2 \delta_2}} + Z_G \right) \\ &- I\left(X_{1D}; \frac{\sqrt{1-\delta_1}}{\sqrt{1 + |h_{21}|^2 \delta_1}} h_{21} X_{1D} + Z_G \right) \\ &+ \frac{1}{2} \log\left(1 + |h_{21}|^2 \delta_1 + |h_{22}|^2 \delta_2\right) - \frac{1}{2} \log(1 + |h_{21}|^2 \delta_1). \end{aligned}$$

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Bounds

Dytso, A.; Tuninetti, D.; Devroye, N., "On discrete alphabets for the two-user Gaussian interference channel with one receiver lacking knowledge of the interfering codebook," ITA, 2014 , vol., no., pp.1,8, 9-14 Feb. 2014

Bounds

Ozarow-Wyner-A

$$I(X_D; \sqrt{\mathrm{S}}X_D + Z) \geq H(X_D) - \mathrm{gap},$$

$$\mathrm{gap} = \xi \log \frac{1}{\xi} + (1-\xi) \log \frac{1}{1-\xi} + \xi \log(N-1) \ ,$$

$$\xi := 2Q\left(\frac{\sqrt{\mathrm{S}} d_{\min(X_D)}}{2}\right),$$

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$$I(X_D; \sqrt{\mathrm{S}} X_D + Z) \geq H(X_D) - \text{gap},$$

$$\text{gap} = \tfrac{1}{2} \log\left(\tfrac{\pi \mathrm{e}}{6}\right) + \tfrac{1}{2} \log\left(1 + \frac{12}{\mathrm{S} d^2_{\min(X_D)}}\right).$$

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DTD-ITA`14-A

$$I(X_D; \sqrt{\mathrm{S}} X_D + Z)$$

$$\geq \left[-\log\left(\sum_{(i,j)\in[1:N]^2} p_i p_j \frac{1}{\sqrt{4\pi}} \mathrm{e}^{-\frac{(s_i - s_j)^2}{4}} \right) - \frac{1}{2} \log(2\pi \mathrm{e}) \right]^+$$

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DTD-ITA`14-B

$$I(X_D; \sqrt{\mathrm{S}} X_D + Z) \geq \log(N) - \mathrm{gap},$$

$$\mathrm{gap} = \frac{1}{2} \log\left(\frac{\mathrm{e}}{2}\right) + \frac{1}{2} \log\left(1 + (N-1)\mathrm{e}^{-\frac{\mathrm{S} d^2_{\min(X_D)}}{4}}\right)$$

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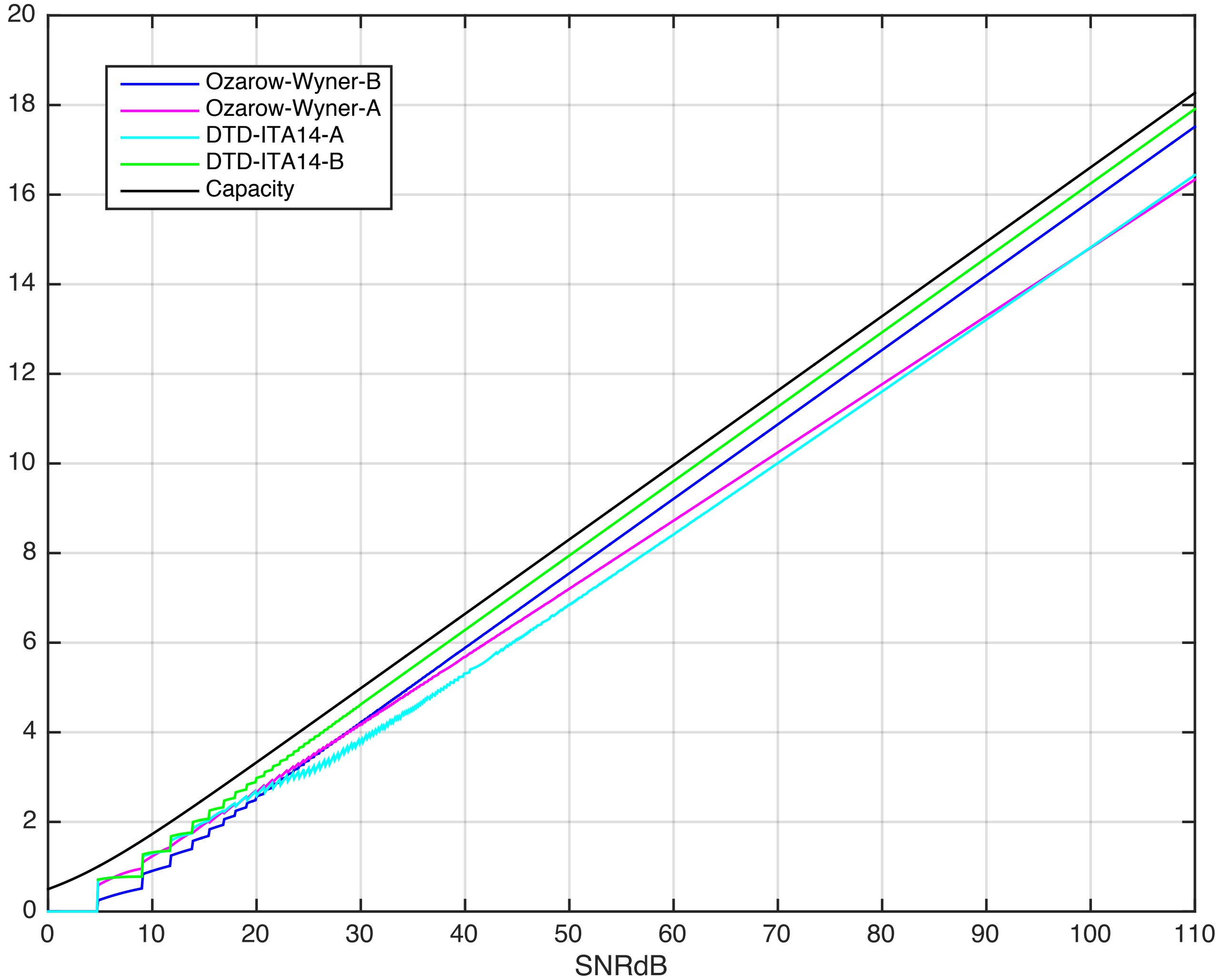
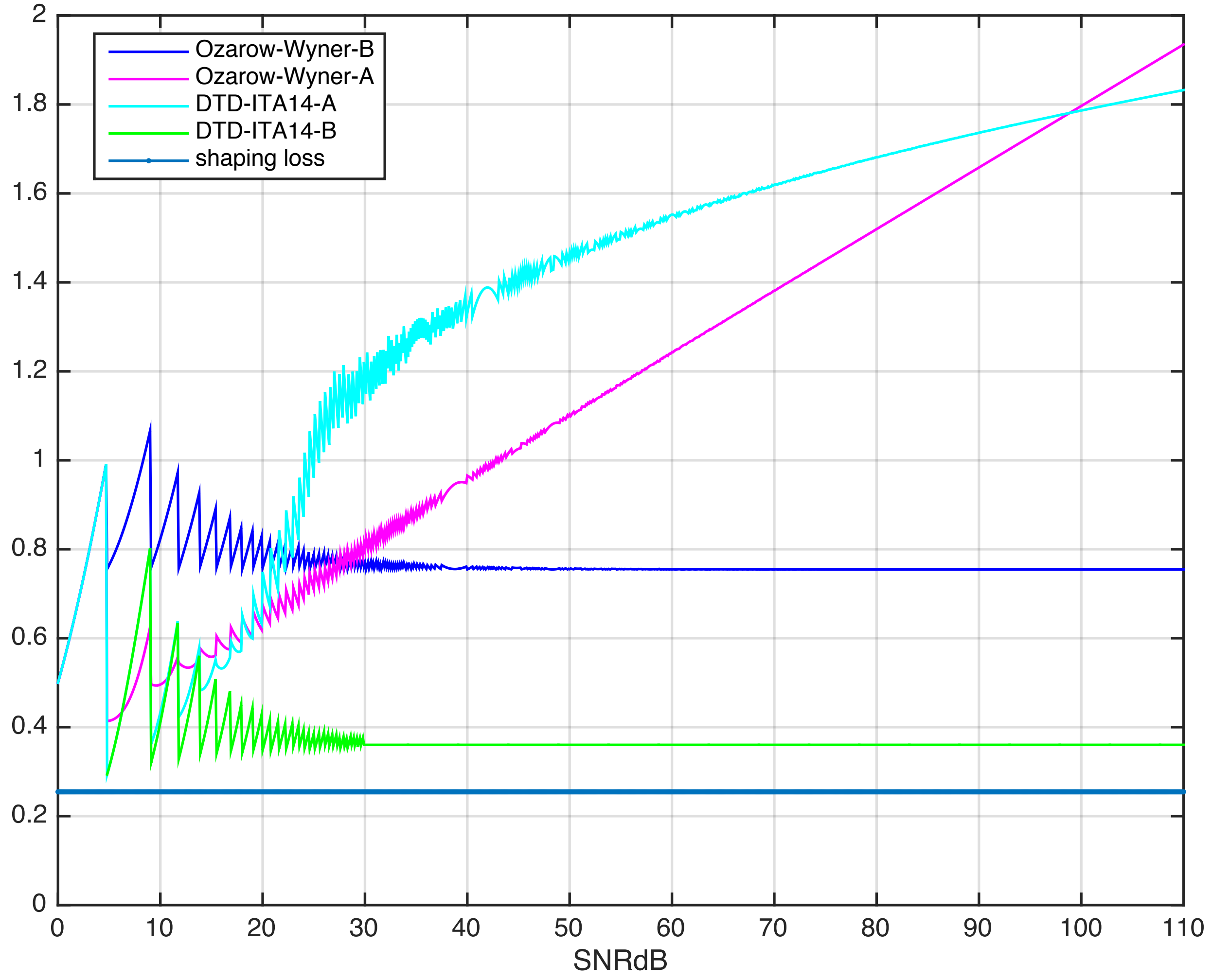
Bound Comparison

Number of Points $N = \lfloor \sqrt{1+S} \rfloor$





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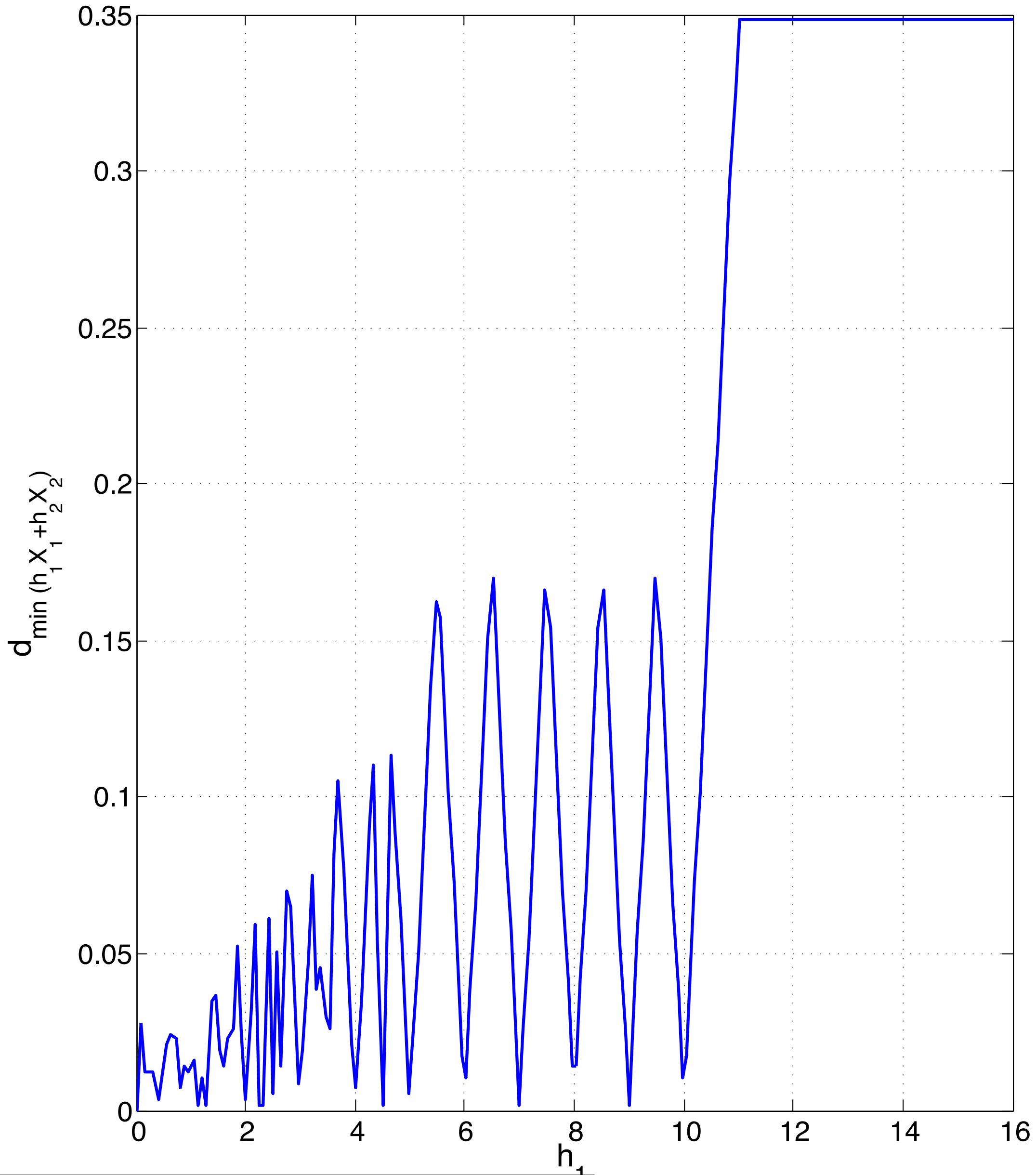
Minimum Distance

Example: h2=1, N1=N2=10



$$I(X_D; \sqrt{\mathrm{S}} X_D + Z) \geq H(X_D) - \text{gap},$$

$$\text{gap} = \frac{1}{2} \log\left(\frac{\pi e}{6}\right) + \frac{1}{2} \log\left(1 + \frac{12}{\mathrm{S} d^2_{\min(X_D)}}\right).$$

$$I(X_{1D}, X_{2D}; h_1 X_{1D} + h_2 X_{2D} + Z)$$

$$d_{\min(h_1 X_{1D} + h_2 X_{2D})}$$

Very Irregular

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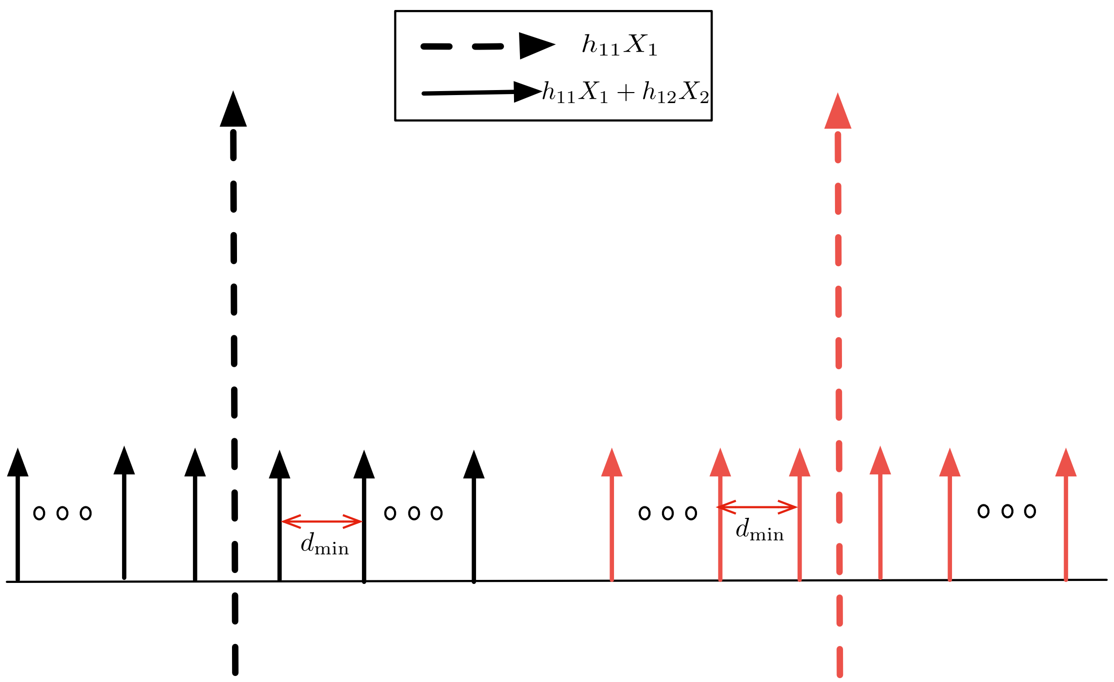
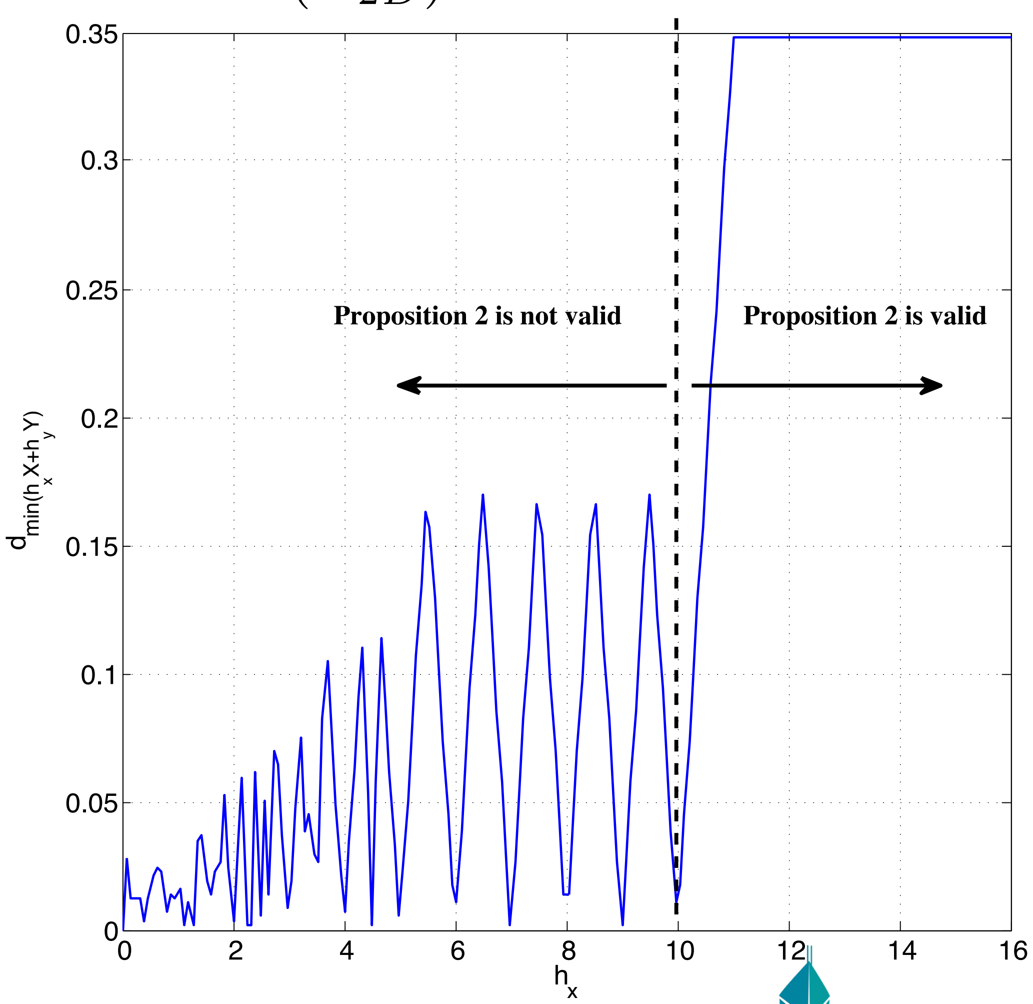
dmin: bound 1

$$d_{\min(h_1 X_{1D}+h_2 X_{2D})} = \min(h_1 d_{\min(X_{1D})}, h_2 d_{\min(X_{2D})})$$
$$\text{if either } h_2 d_{\min(X_{2D})} N_2 \le h_1 d_{\min(X_{1D})}$$
$$\text{or } h_1 d_{\min(X_{1D})} N_1 \le h_2 d_{\min(X_{2D})}$$



No Overlaps



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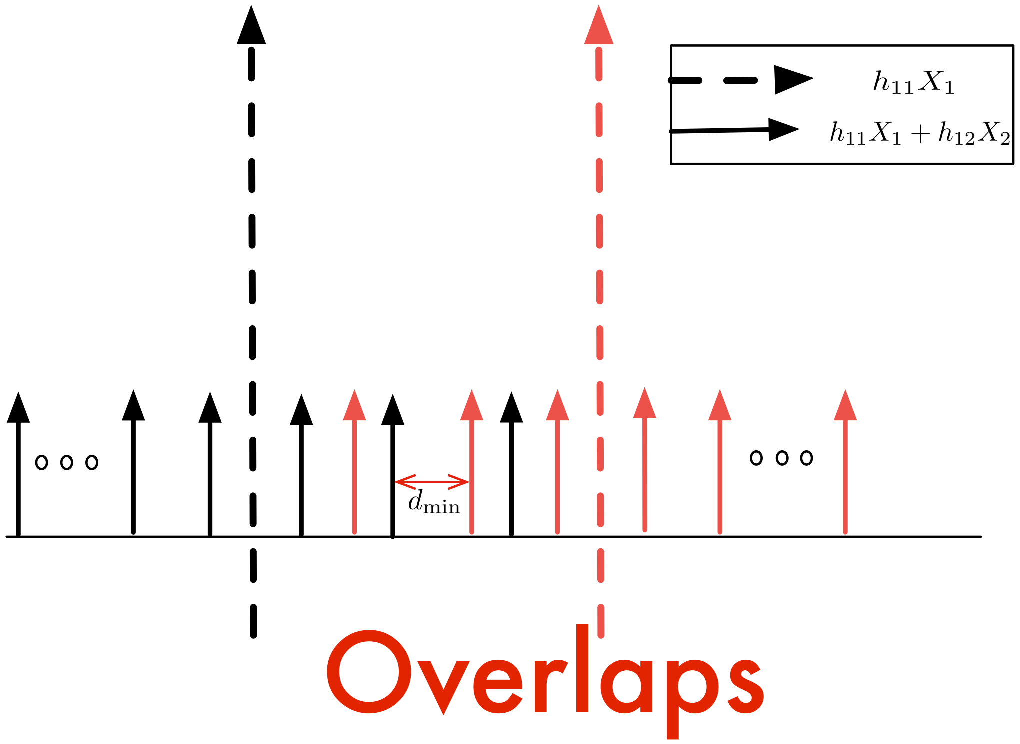
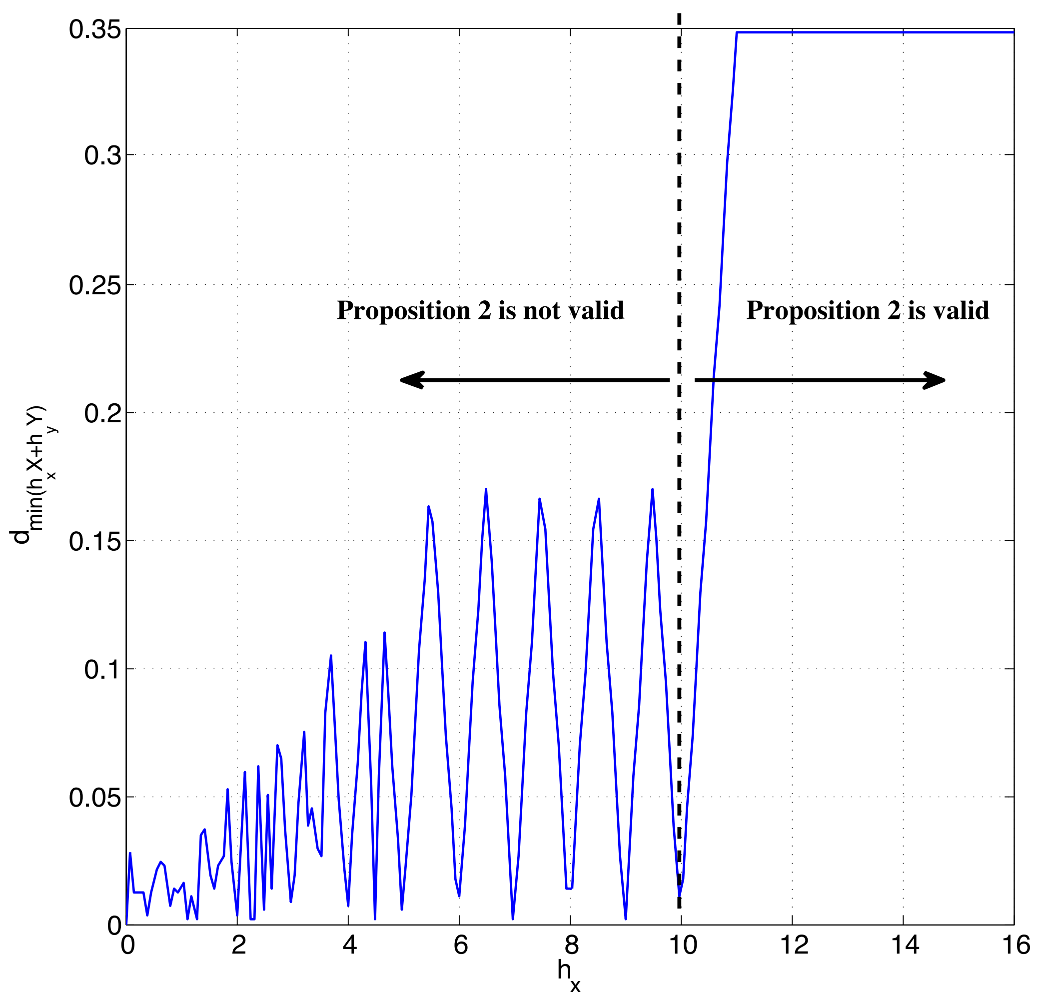
dmin: bound 2

$$d_{\min(h_1X_{1D}+h_2X_{2D})} = \kappa \min\left(h_1 d_{\min(X_{1D})}, h_2 d_{\min(X_{2D})}, \max\left(\frac{h_1 d_{\min(X_{1D})}}{N_2}, \frac{h_2 d_{\min(X_{2D})}}{N_1}\right)\right)$$

$$\kappa = \frac{\gamma}{(1+\log(\max(N_1,N_2)))}$$

for all (h_1, h_2) except an outage set of measure γ for any $\gamma > 0$.





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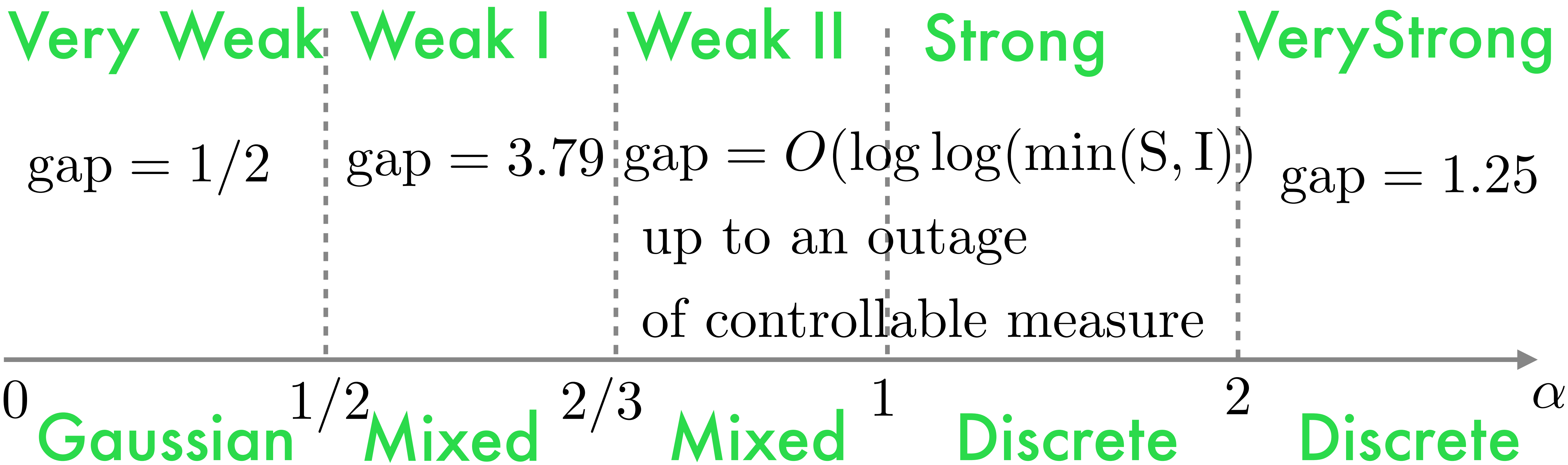
Main Result



$$X_i = \sqrt{1-\delta_i}\; X_{iD} + \sqrt{\delta_i}\; X_{iG},\;\; i \in [1:2],$$

$$\alpha = \frac{\log \mathrm{I}}{\log \mathrm{S}}$$

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Main Result

Very Weak	Weak I	Weak II	Strong	VeryStrong
$\text{gap} = 1/2$	$\text{gap} = 3.79$	$\text{gap} = O(\log\log(\min(\mathrm{S},\mathrm{I})))$ up to an outage of controllable measure		$\text{gap} = 1.25$
$0 \le \alpha \le 1/2$	$1/2 \le \alpha \le 2/3$	$2/3 \le \alpha \le 1$	$1 \le \alpha \le 2$	$\alpha \ge 2$
Gaussian	Mixed	Mixed	Discrete	Discrete

$$X_i = \sqrt{1-\delta_i}\; X_{iD} + \sqrt{\delta_i}\; X_{iG}, \; i \in [1:2],$$

$$\alpha = \frac{\log \mathrm{I}}{\log \mathrm{S}}$$

Closed-form expressions for number of points, power splits and gap

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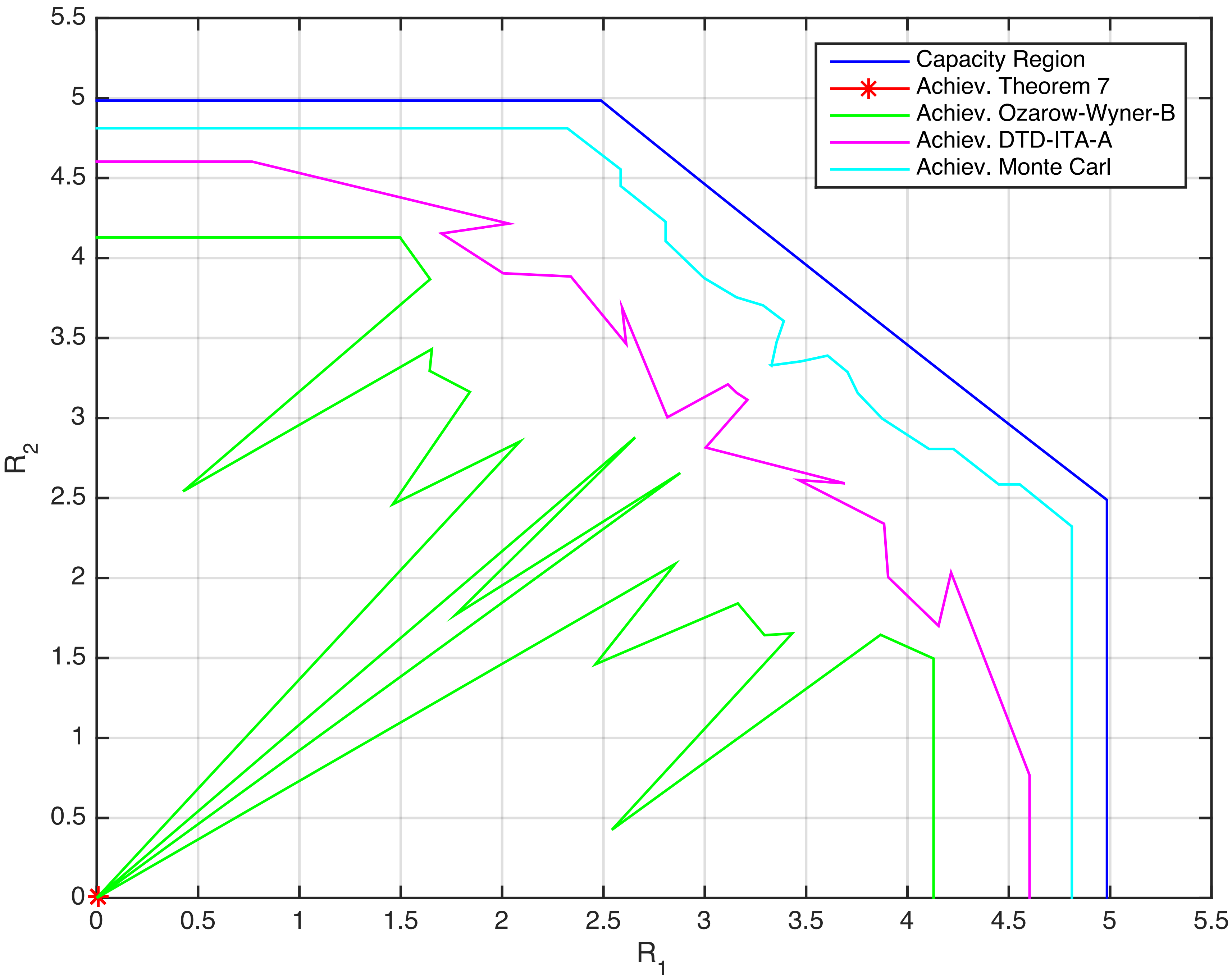
Example



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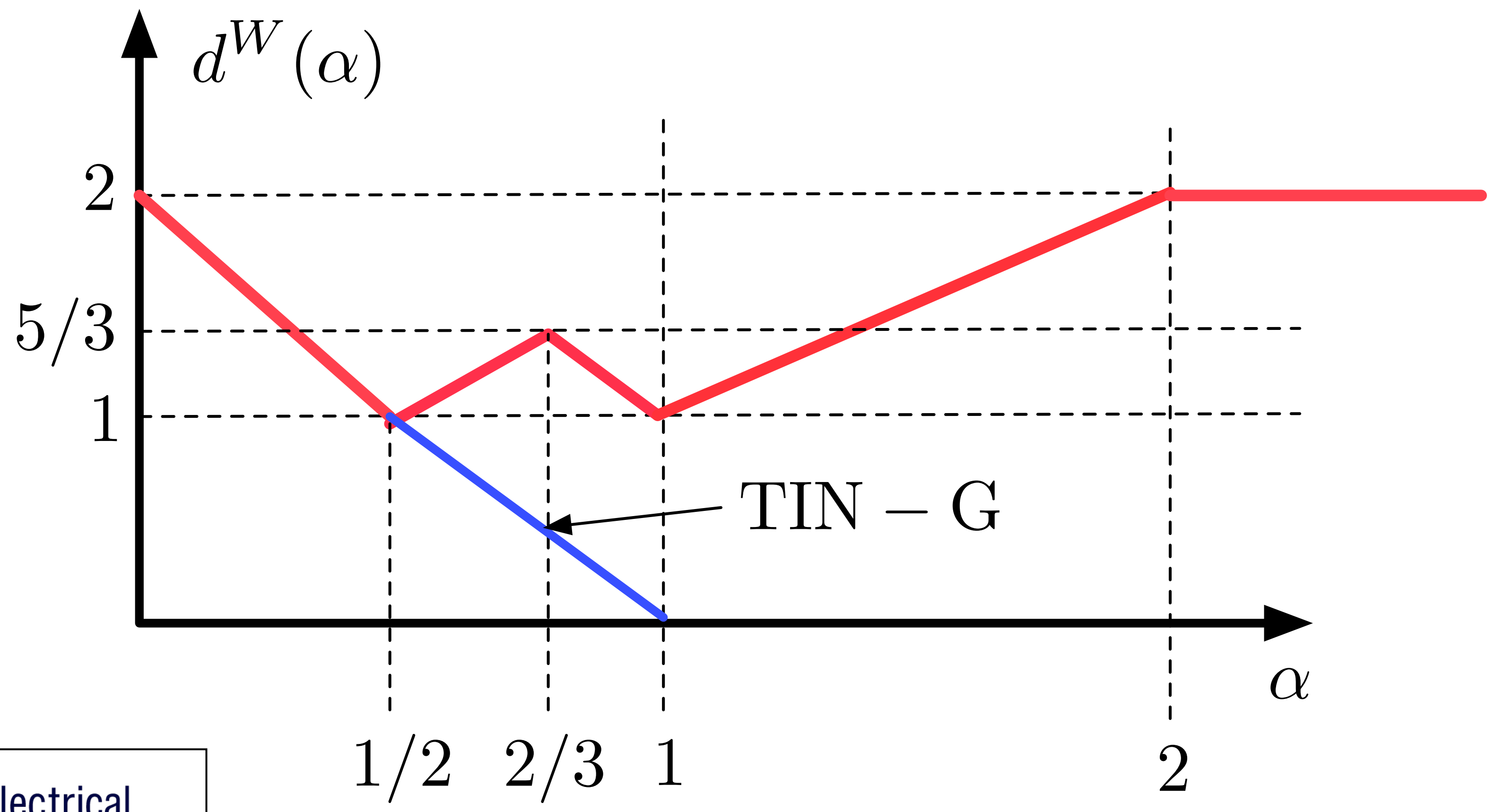
gDoF Optimality

<u>Main result ISIT` 15:</u>

TINnoTS is gDoF optimal up to a set of zero measure.



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Concluding Remarks

- Key idea: use non-Gaussian inputs
- Developed very general tools of use beyond 2-IC
- Applicable to Block Asynchronous Interference Channel and Codebook Oblivious Interference Channel

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Thank you

arXiv:1506.02597

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Thank you

arXiv:1506.02597

Gap Result

Theorem: TINnoTS is within a gap of the capacity given by:

- Very Weak Interference: $\mathrm{S} \geq \mathrm{I}(1+\mathrm{I})$: gap $\leq \frac{1}{2}$ bits,
- Moderately Weak Interference Type2: $\mathrm{S} < \mathrm{I}(1+\mathrm{I})$, $\frac{1+\mathrm{S}}{1+\mathrm{I}+\frac{\mathrm{S}}{1+\mathrm{I}}} > \frac{1+\mathrm{I}+\frac{\mathrm{S}}{1+\mathrm{I}}}{1+\frac{\mathrm{S}}{1+\mathrm{I}}}$:
  gap $\leq \frac{1}{2}\log\left(\frac{608\ \pi e}{27}\right) \approx 3.79$bits
- Moderately Weak Interference Type1: $\mathrm{I} \leq \mathrm{S}$, $\frac{1+\mathrm{S}}{1+\mathrm{I}+\frac{\mathrm{S}}{1+\mathrm{I}}} \leq \frac{1+\mathrm{I}+\frac{\mathrm{S}}{1+\mathrm{I}}}{1+\frac{\mathrm{S}}{1+\mathrm{I}}}$:
  gap $\leq \frac{1}{2}\log\left(\frac{16\pi e}{3}\right) + \frac{1}{2}\log\left(1 + 45 \cdot \frac{(1+1/2\ln(1+\min(\mathrm{I},\mathrm{S})))^2}{\gamma^2}\right)$ bits, except for a set of measure γ for any $\gamma \in (0,1]$,
- Strong Interference: $\mathrm{S} < \mathrm{I} < \mathrm{S}(1+\mathrm{S})$:
  gap $\leq \frac{1}{2}\log\left(\frac{2\pi e}{3}\right) + \frac{1}{2}\log\left(1 + 8 \cdot \frac{(1+1/2\ln(1+\min(\mathrm{I},\mathrm{S})))^2}{\gamma^2}\right)$ bits,
  except for a set of measure γ for any $\gamma \in (0,1]$,
- Very Strong Interference: $\mathrm{I} \geq \mathrm{S}(1+\mathrm{S})$:
  gap $\leq \frac{1}{2}\log\left(\frac{2\pi e}{3}\right) \approx 1.25$ bits.